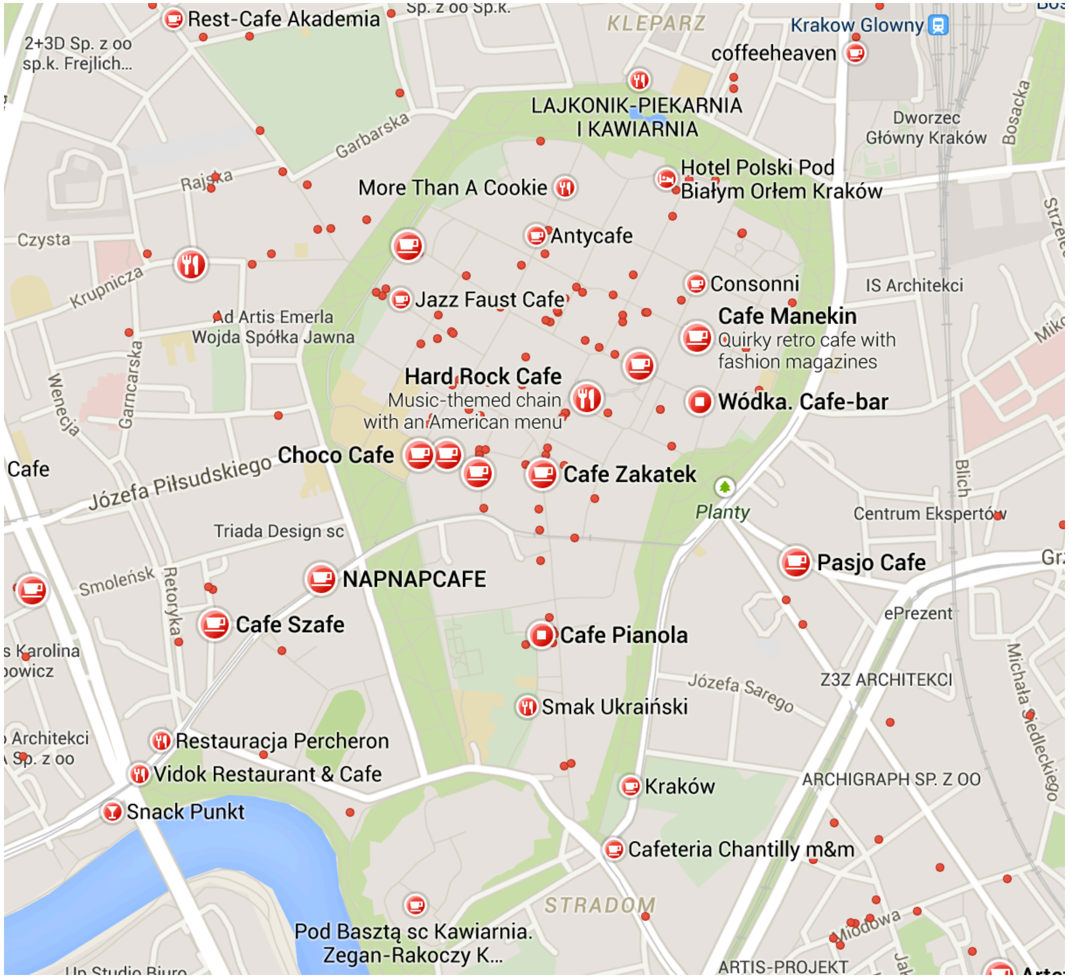
Drift-diffusion model of decision making

Anatoly Buchin & Boris Gutkin

27/03/2015

Choose where to go on Friday night

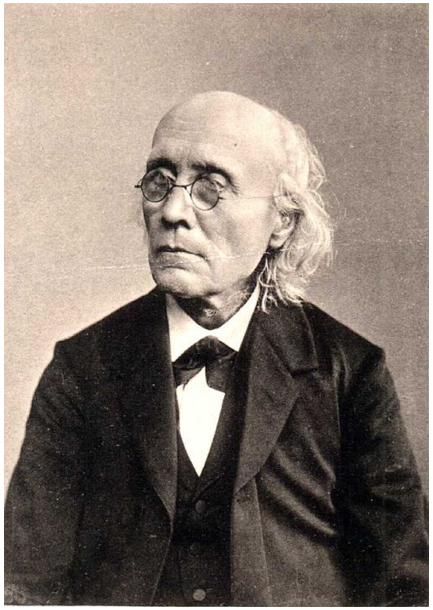


Alternative Forced Choice task (AFC-task)

- Psychophysical method to test an experience of a stimulus based on expectation

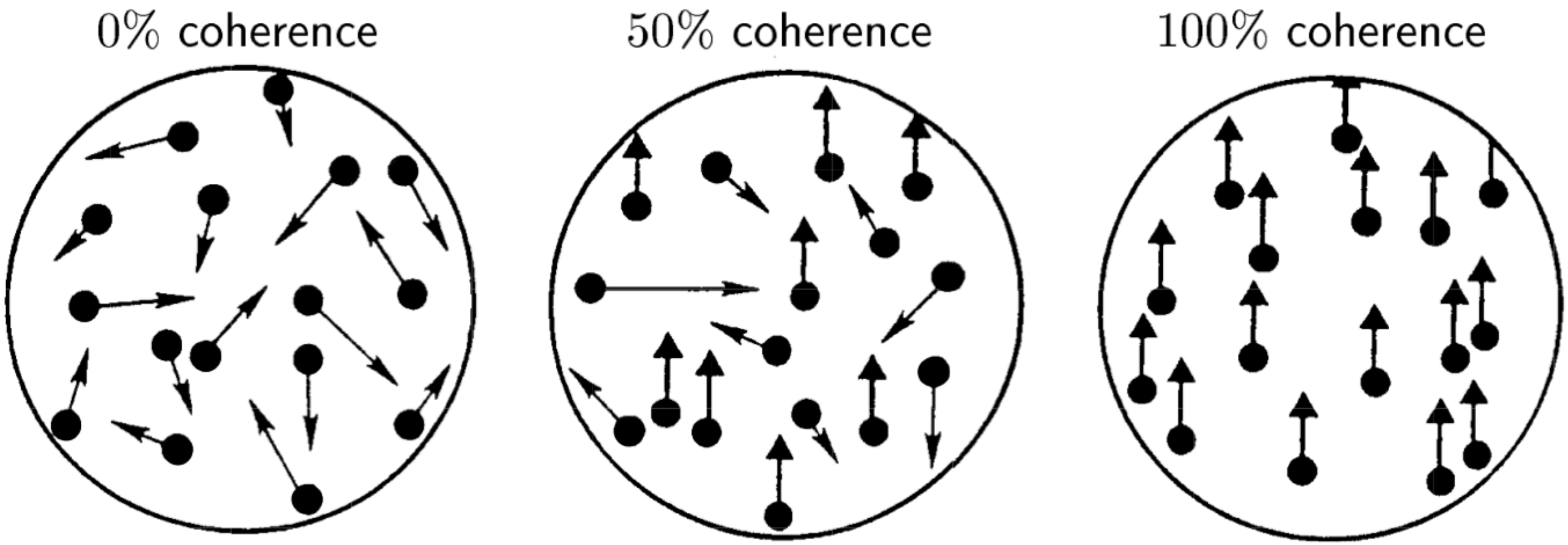
- Basic concept:
- two alternative choices present simultaneously
- delay interval to make the choice possible
- Response indicating choice of a stimuli (obligatory)

- What could be studied?
- Speed of choice
- Accuracy of selection
- Ambiguity of a stimulus



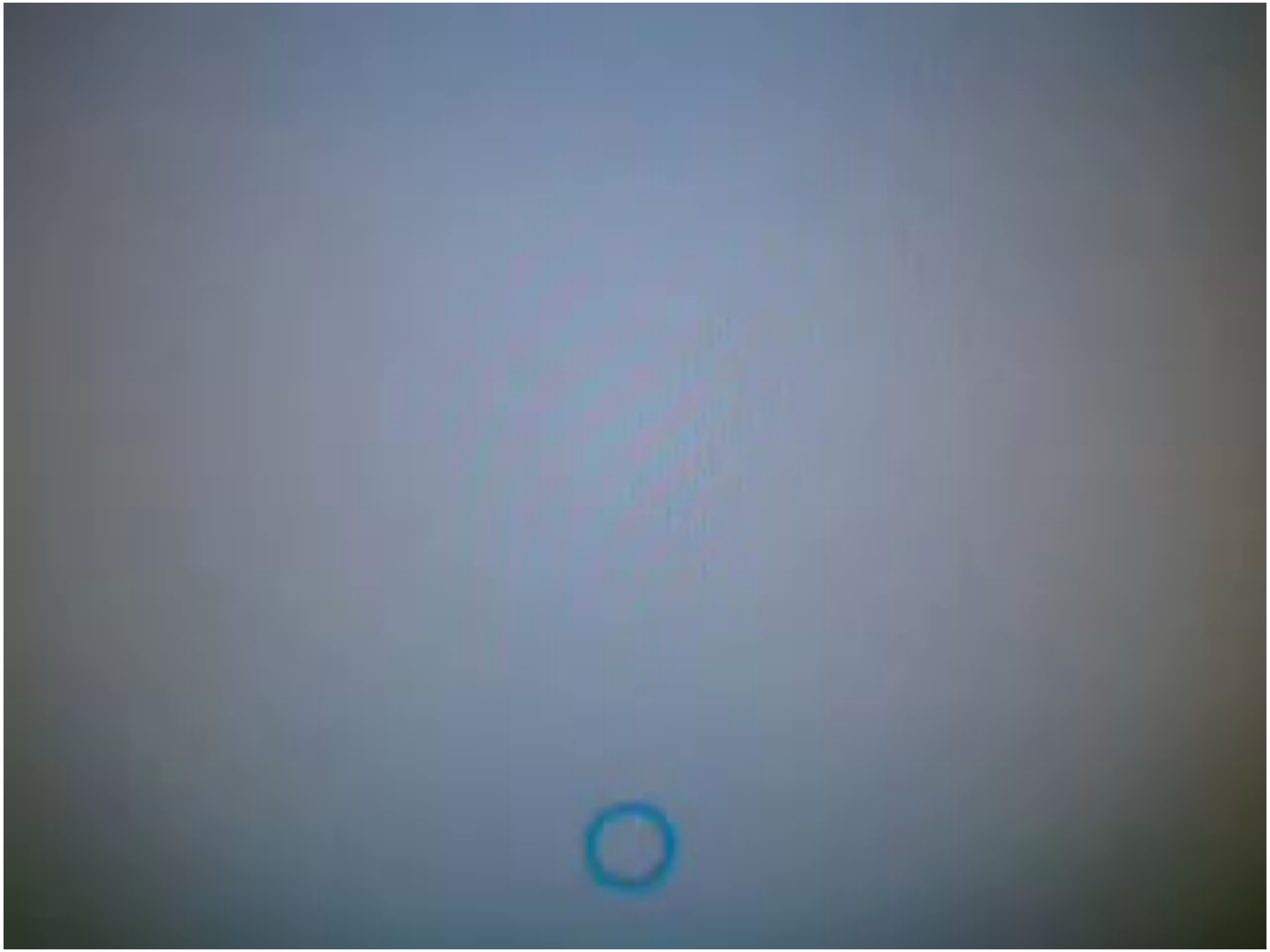
Gustav Theodor Fechner
1801 - 1887

Example of AFC-task: Visual stimuli with different stimulus coherence

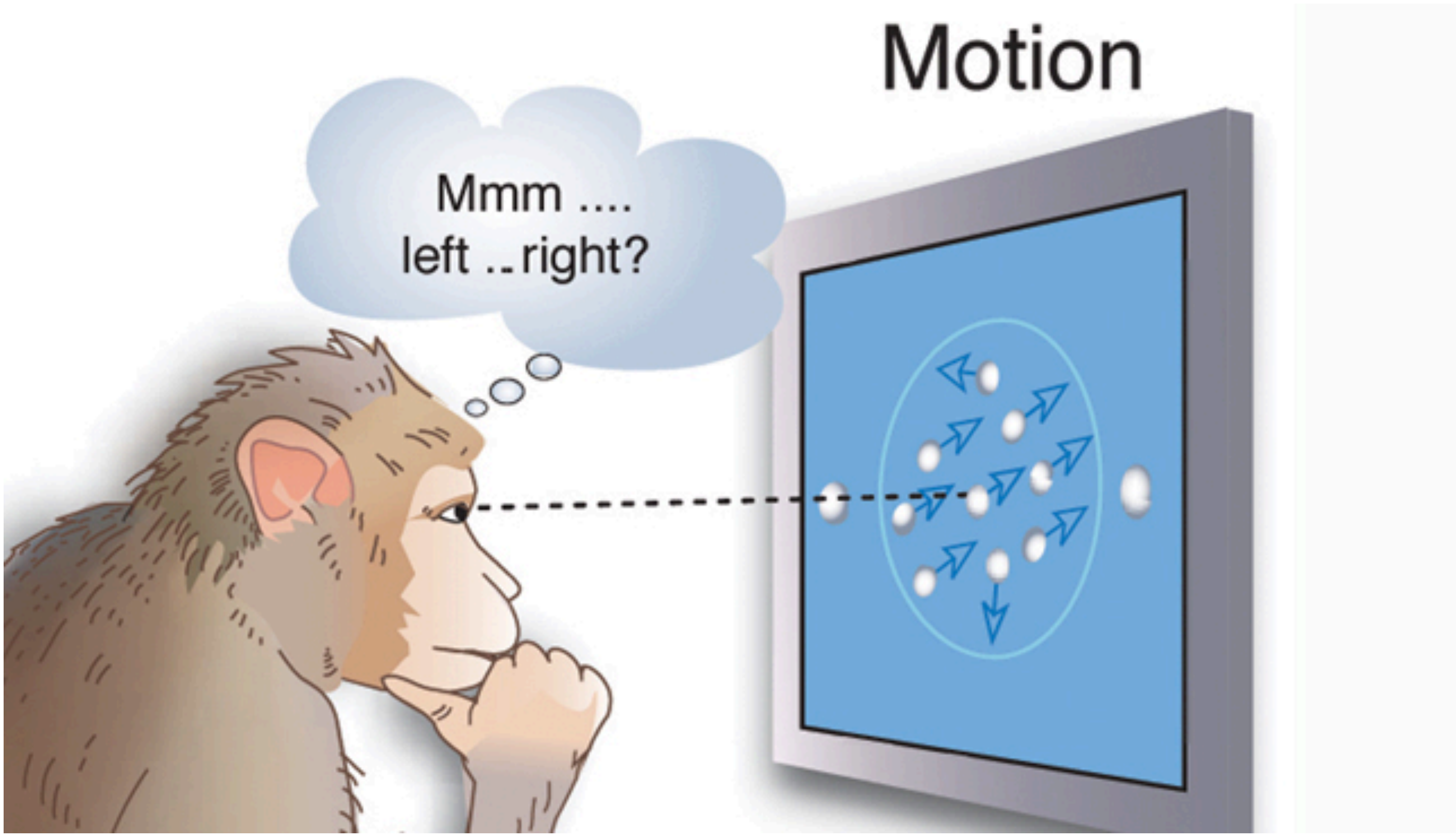


[Dayan and Abott, 2001]

Visual stimulus for a given level of coherence

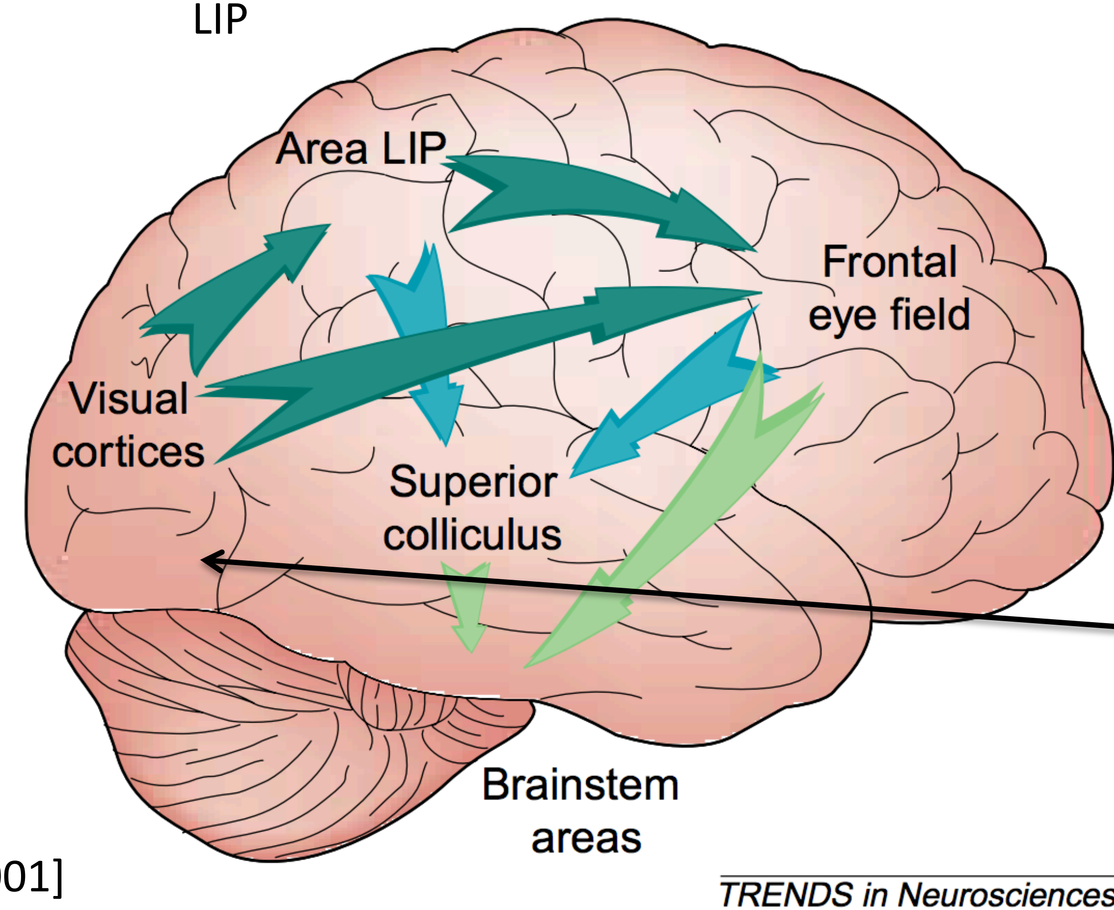


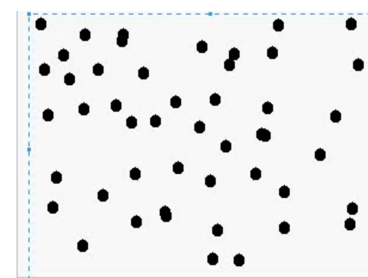
Choice based on coherence



Saccadic decision making model

Lateral Intraparietal Cortex
LIP

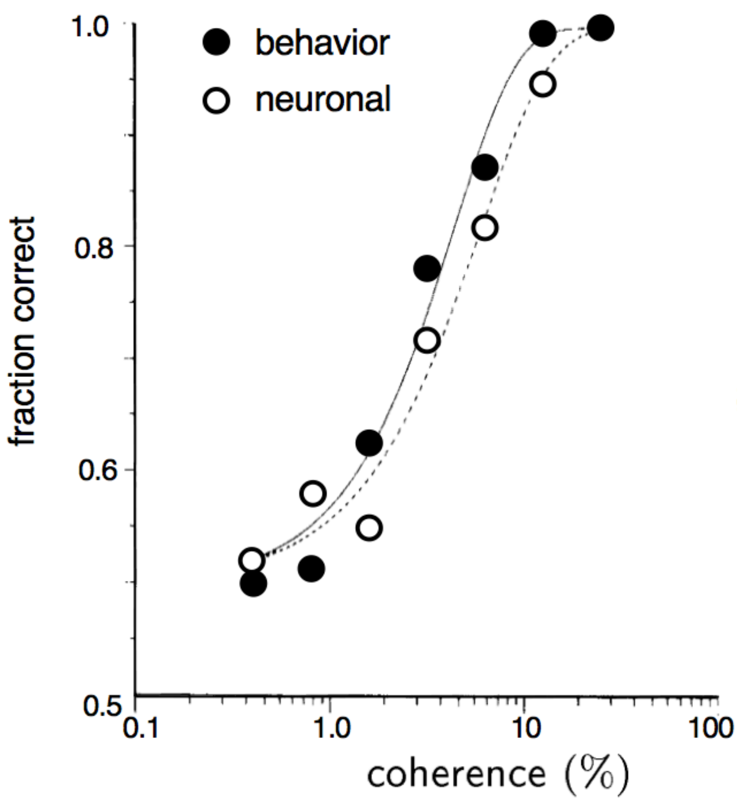


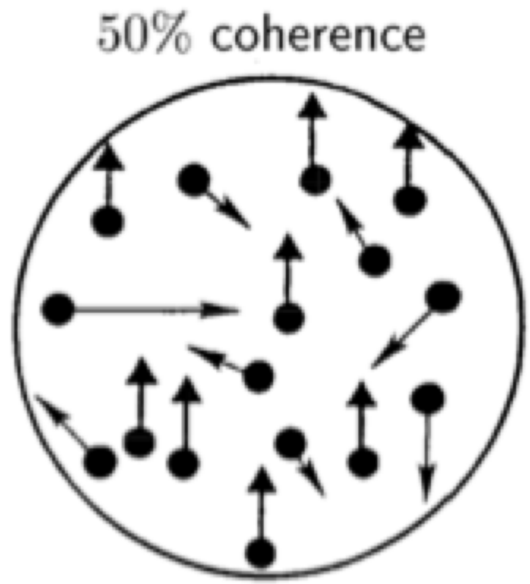


[Glimcher 2001]

Choice based on coherence

A [Dayan and Abott, 2001]



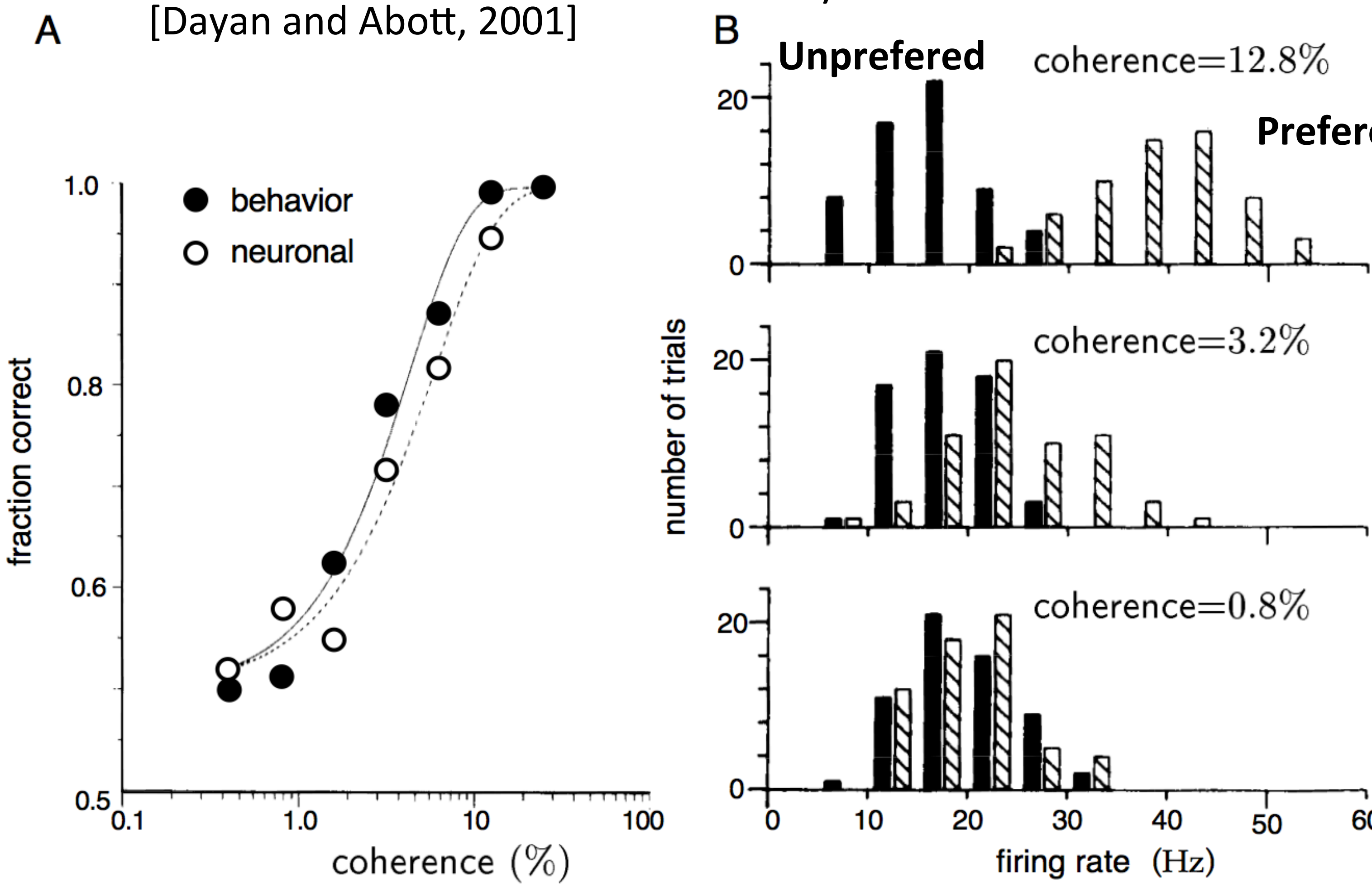


Choice based on coherence

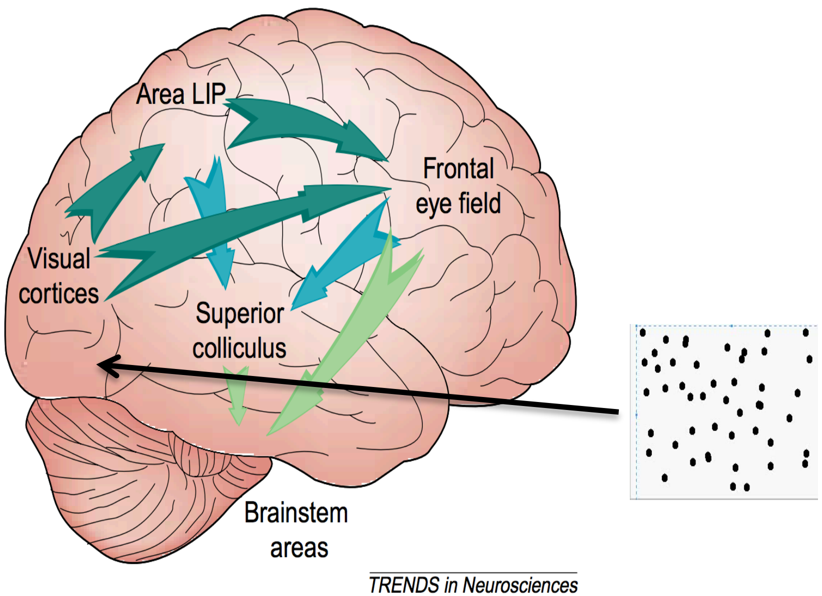
Neural correlate

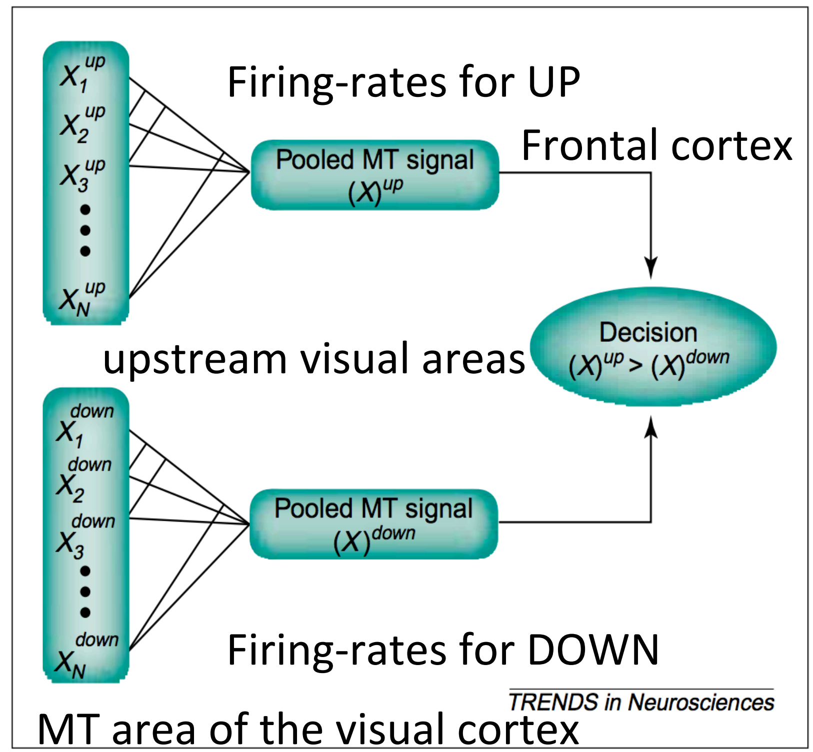
activity in the MT area of the visual cortex

[Dayan and Abott, 2001]



Drift-diffusion model of decision making

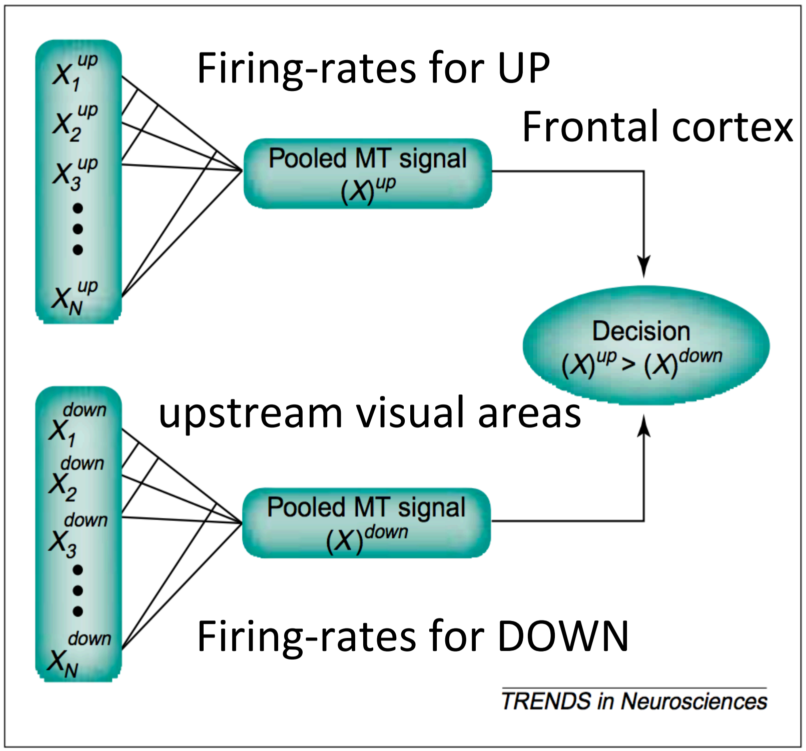


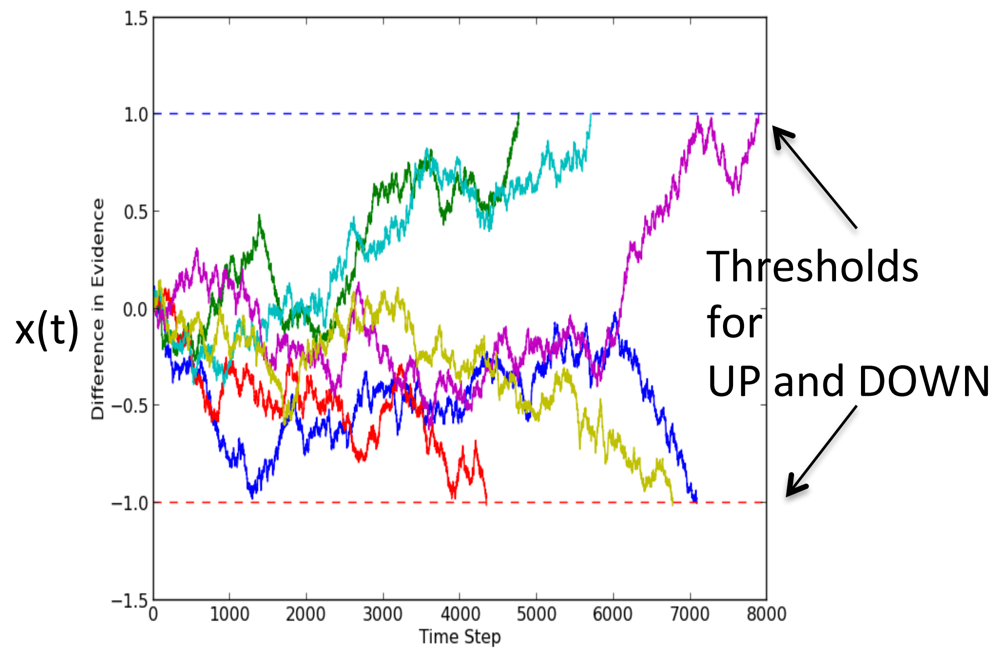


[Shalden et al, 1996]

[Glimcher 2001]

Drift-diffusion model of decision making





MT area of a visual cortex

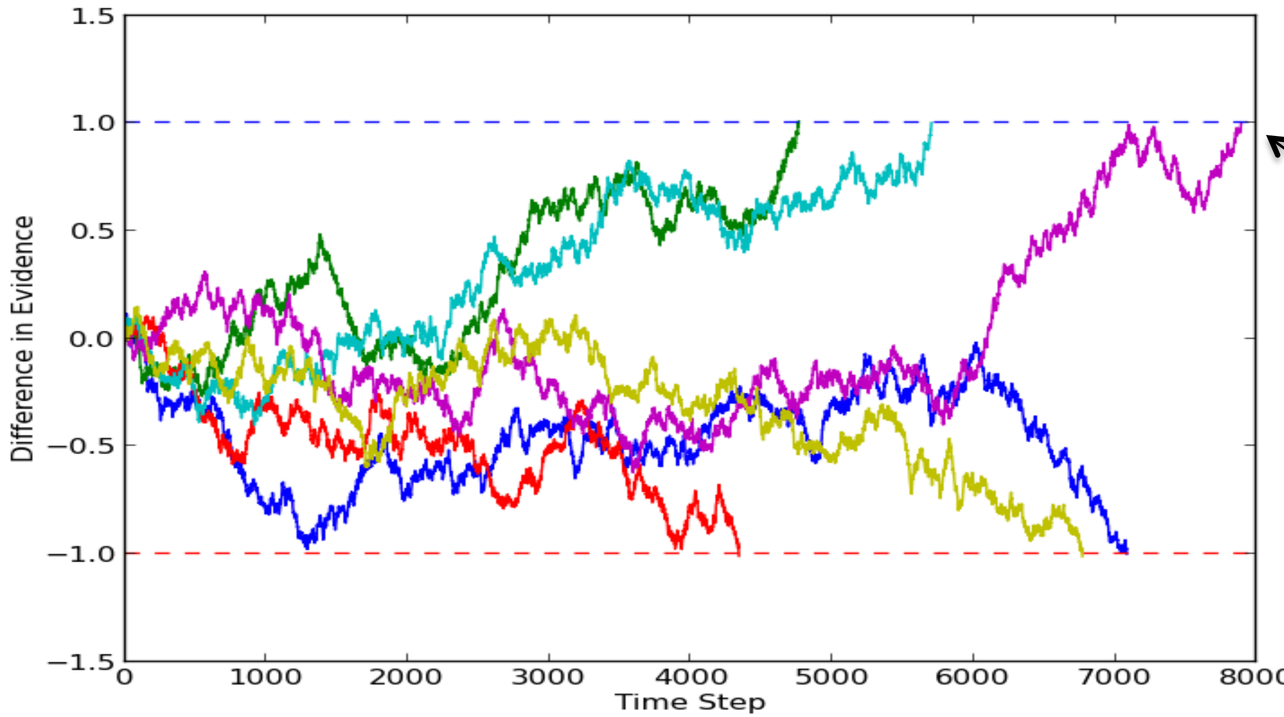
Drift-diffusion model of decision making: mathematical formulation

$$\frac{dx}{dt} = m_A - m_B + \sigma\eta(t)$$

Firing-rates for UP and DOWN

Stimulus/neural noise

x(t)



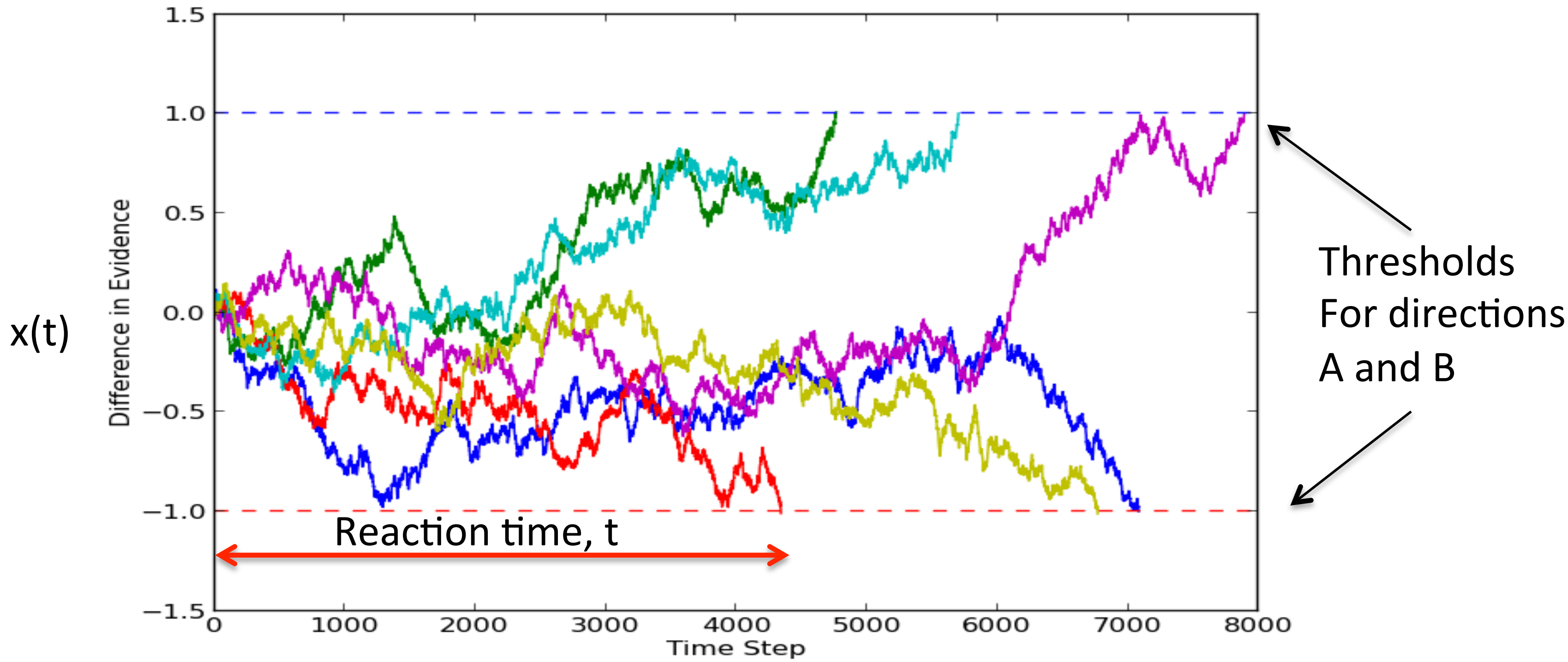
Thresholds
For directions
A and B

Drift-diffusion model of decision making: mathematical formulation

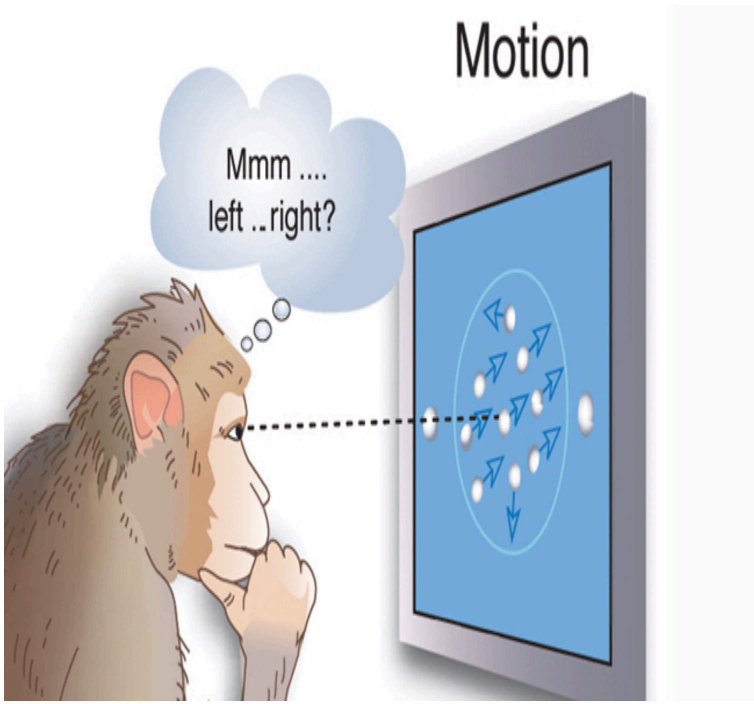
$$\frac{dx}{dt} = m_A - m_B + \sigma\eta(t)$$

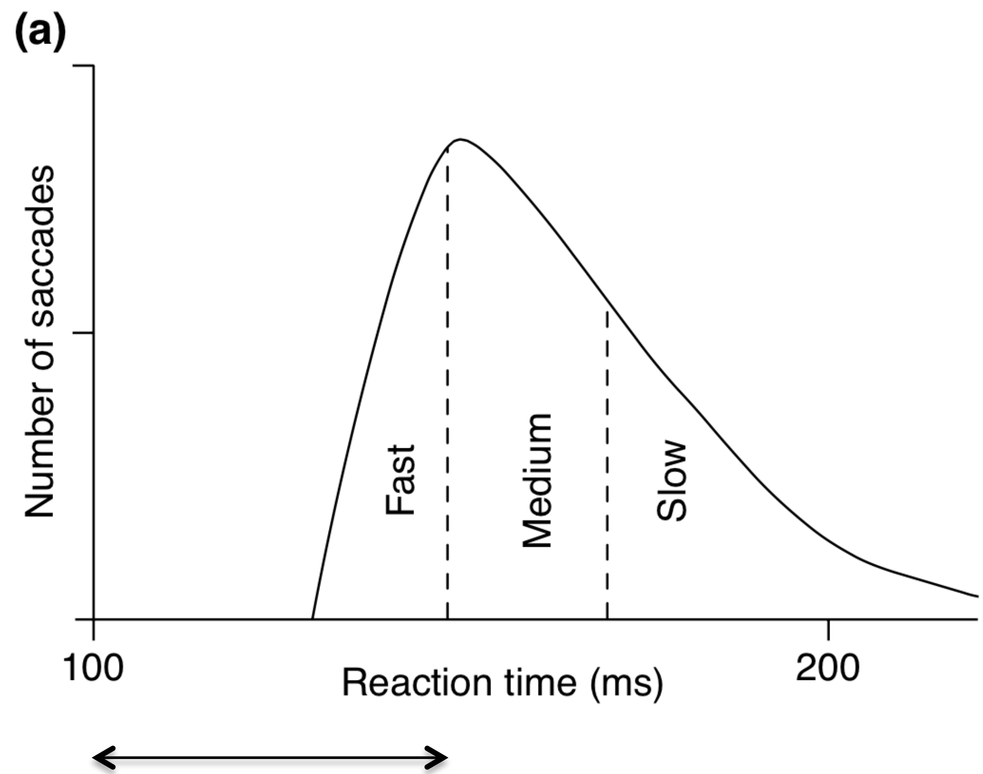
Firing-rates for UP and DOWN

Stimulus/neural noise



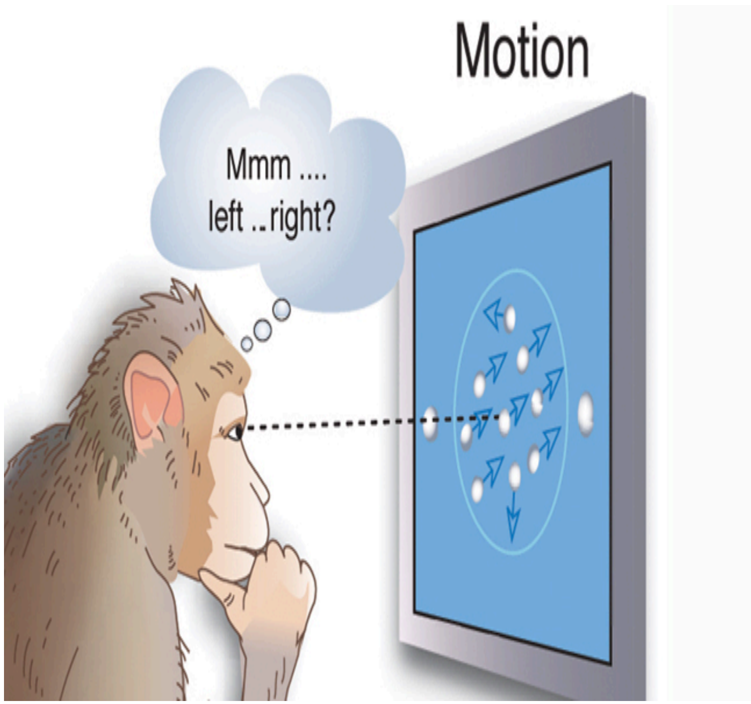
Reaction time

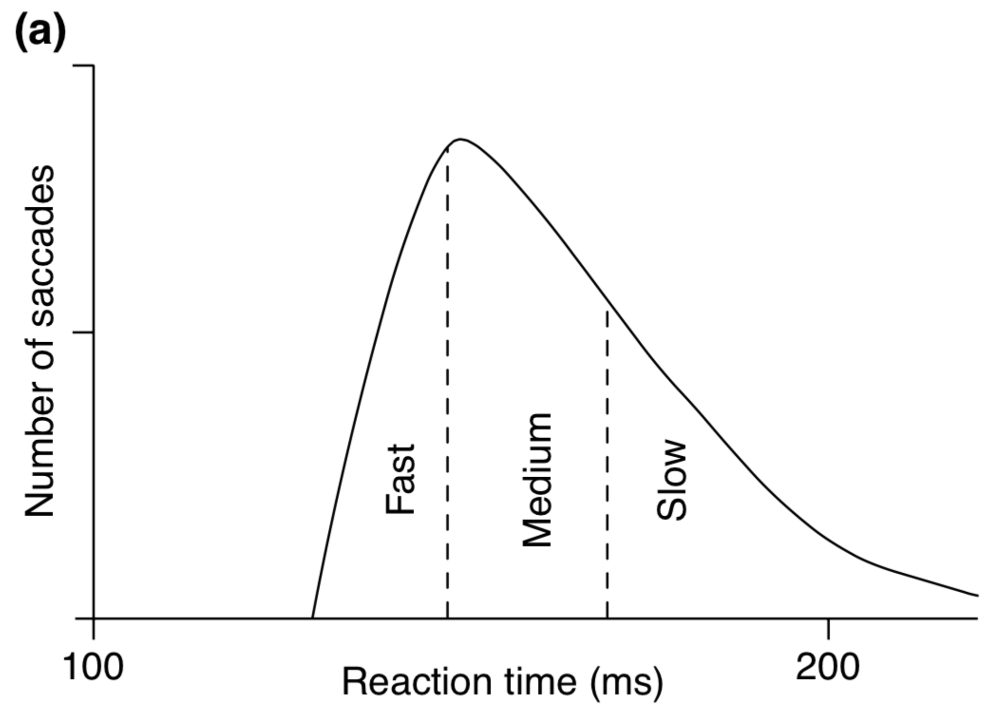




T ~ 100ms

Reaction time





T ~ 100ms

t – reaction time from Drift-Diffusion model

Total reaction time = T+ t

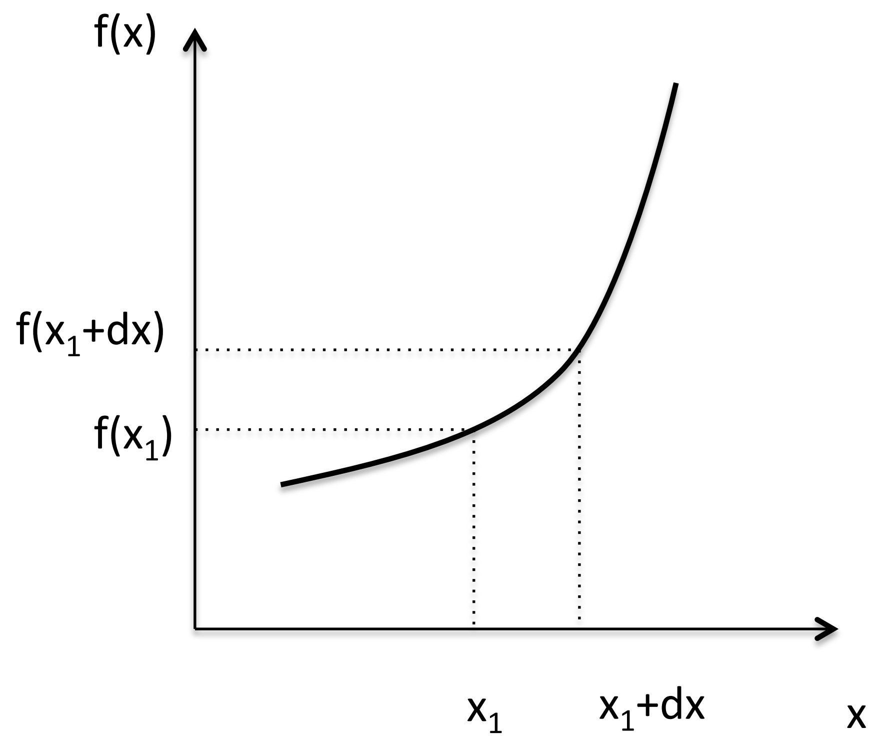
How to integrate the differential equation?

$$\frac{dx}{dt} = m_A - m_B + \sigma\eta(t)$$

$$\Updownarrow$$

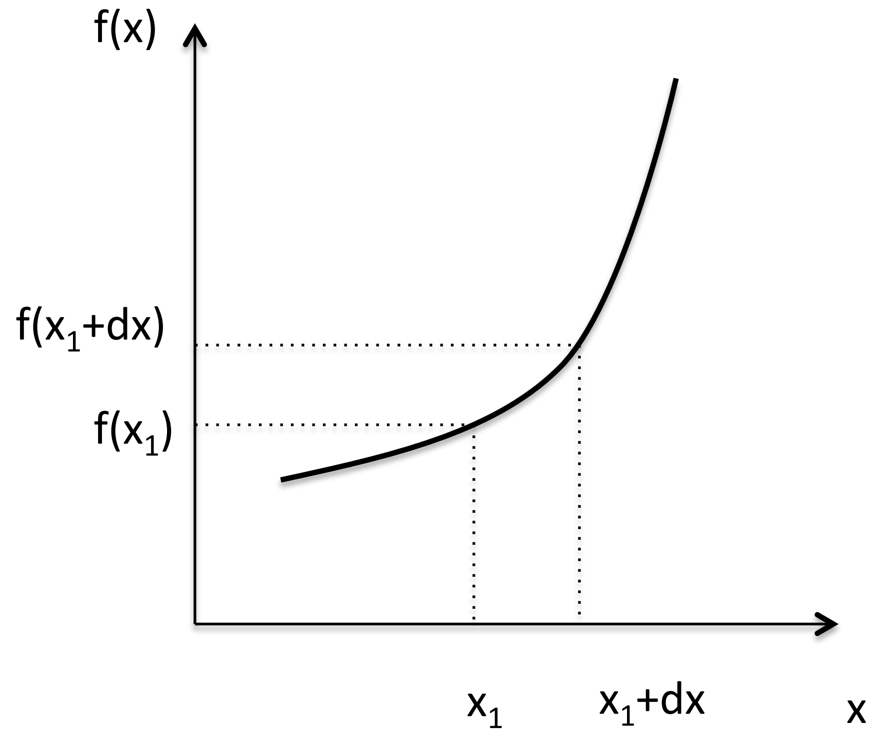
$$\frac{dx}{dt} = f(x)$$

Math reminder: derivative



Math reminder: derivative

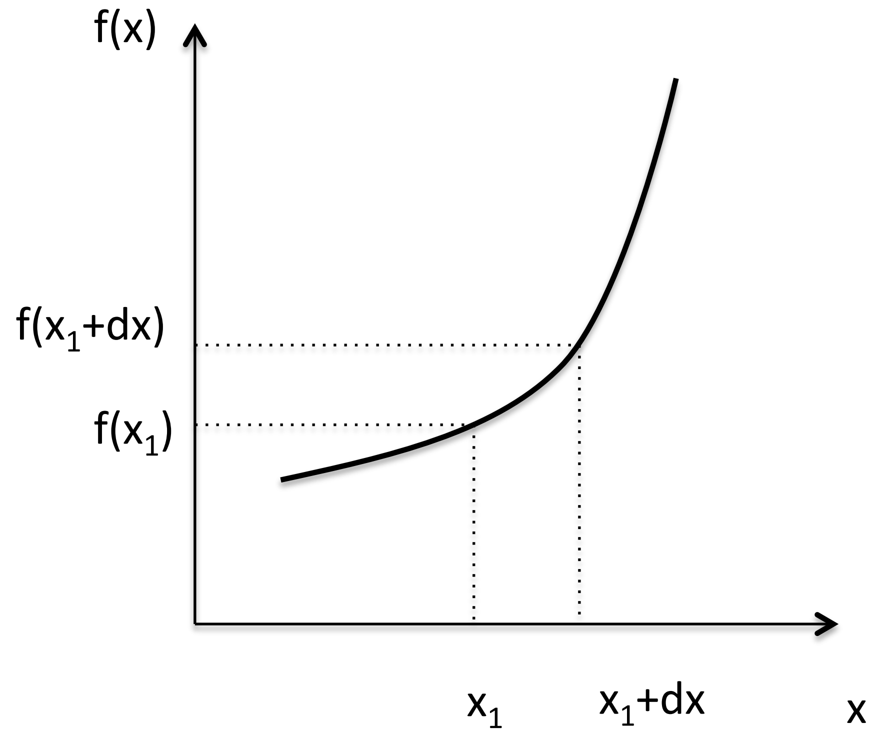
$$\frac{f(x_1 + dx) - f(x_1)}{x_1 + dx - x_1}$$



Math reminder: derivative

$$\frac{f(x_1 + dx) - f(x_1)}{x_1 + dx - x_1}$$

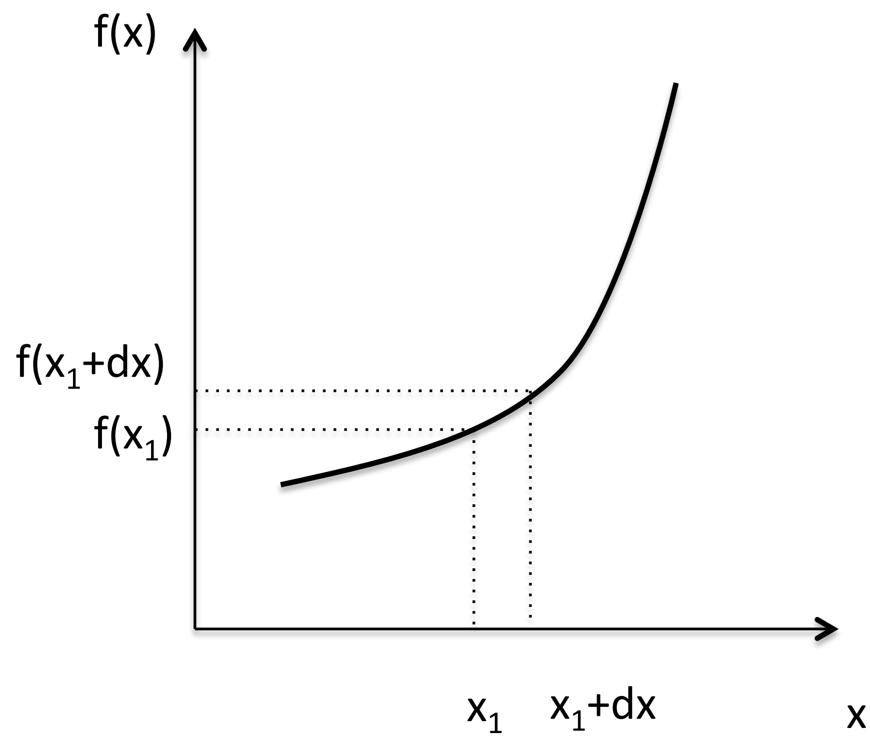
$$\frac{f(x_1 + dx) - f(x_1)}{dx}$$



Math reminder: derivative

$$\frac{f(x+dx)-f(x)}{dx}$$

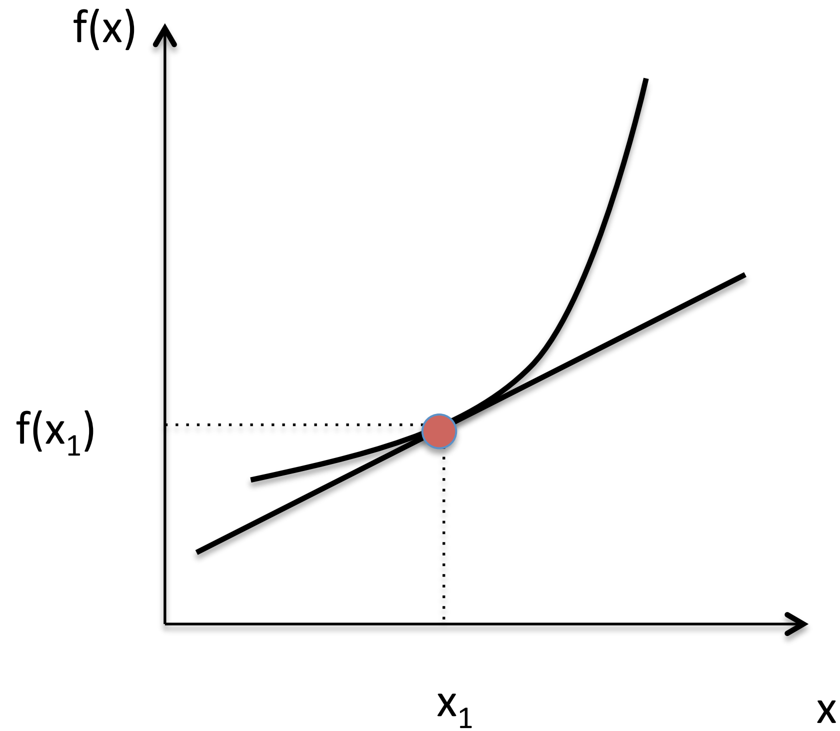
$$\lim_{dx \to 0} \frac{f(x_1+dx)-f(x_1)}{dx}$$



Math reminder: derivative

$$\frac{f(x+dx)-f(x)}{dx}$$

$$\lim_{dx\to 0}\frac{f(x_1+dx)-f(x_1)}{dx}=\frac{df(x)}{dx}$$



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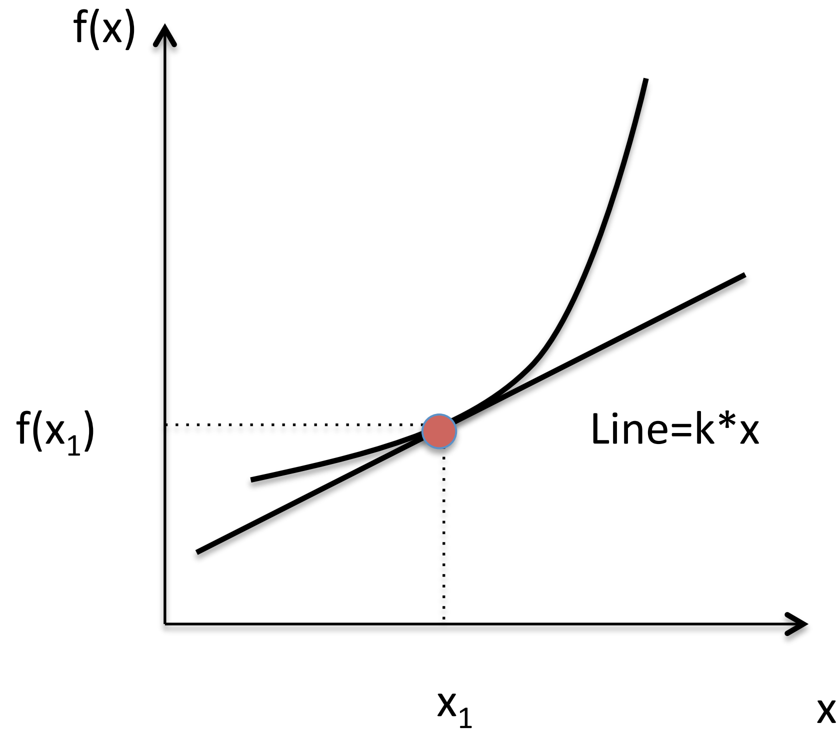
$$\left.\frac{df(x)}{dx}\right|_{x_1}=k \qquad speed$$



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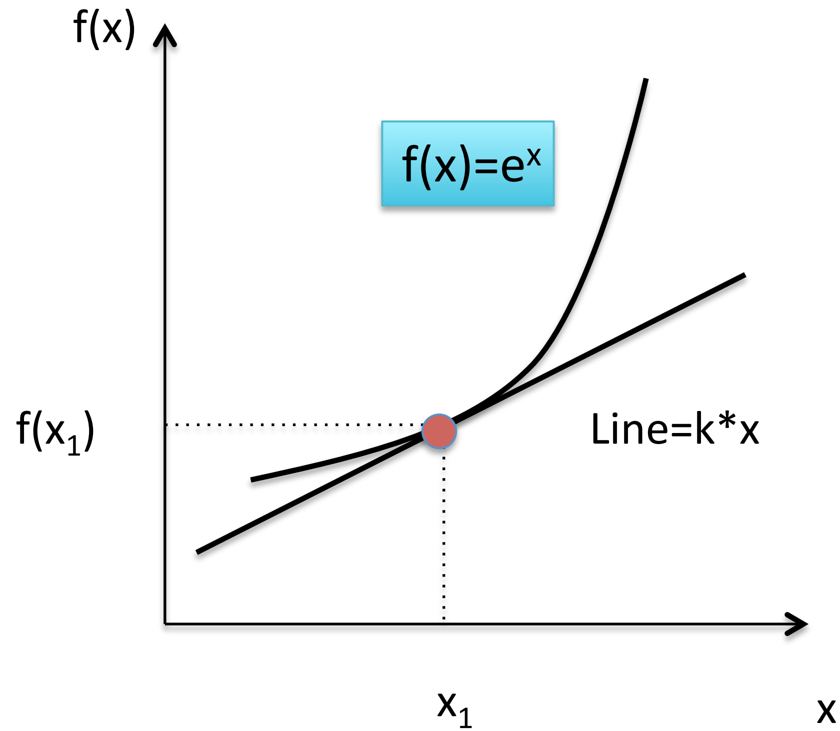

Table of Standard Differential Coefficients

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
x^n	nx^{n-1}		
e^x	e^x	a^x	$a^x \ln a \qquad (a>0)$
$\ln x$	$\frac{1}{x} \qquad (x>0)$		
$\sin x$	$\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$-\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$\sec^2 x$	$\tanh x$	$\text{sech}^2 x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$	$\text{cosech}\, x$	$-\text{cosech}\, x \coth x$
$\sec x$	$\sec x \tan x$	$\text{sech}\, x$	$-\text{sech}\, x \tanh x$
$\cot x$	$-\text{cosec}^2 x$	$\coth x$	$-\text{cosech}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}} \qquad (-1<x<1)$	$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}} \qquad (-1<x<1)$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}} \qquad (x>1)$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\tanh^{-1} x$	$\frac{1}{1-x^2} \qquad (-1<x<1)$

Euler method of integration of differential equation

$$\frac{dx}{dt} = m_A - m_B + \sigma\eta(t)$$

$$\frac{dx}{dt} = f(x)$$

Euler method of integration of the differential equation

$$\frac{dx}{dt} = f(x)$$

$$\frac{dx}{dt} \stackrel{def}{=} \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Euler method of integration of the differential equation

$$\frac{dx}{dt} = f(x)$$

$$\frac{dx}{dt} \stackrel{def}{=} \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$small$$

$$\Delta t$$

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Euler method of integration of the differential equation

$$\frac{dx}{dt} = f(x)$$

$$\frac{dx}{dt} \stackrel{def}{=} \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t} = f(x)$$

$small$

Δt

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Euler method of integration of the differential equation

$$\frac{dx}{dt} = f(x)$$

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$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t} = f(x)$$

small

Δt

$$x(t+\Delta t) - x(t) = f(x)\Delta t$$

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Euler method of integration of the differential equation

$$\frac{dx}{dt} = f(x)$$

$$\frac{dx}{dt} \stackrel{def}{=} \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t} = f(x)$$

small

Δt

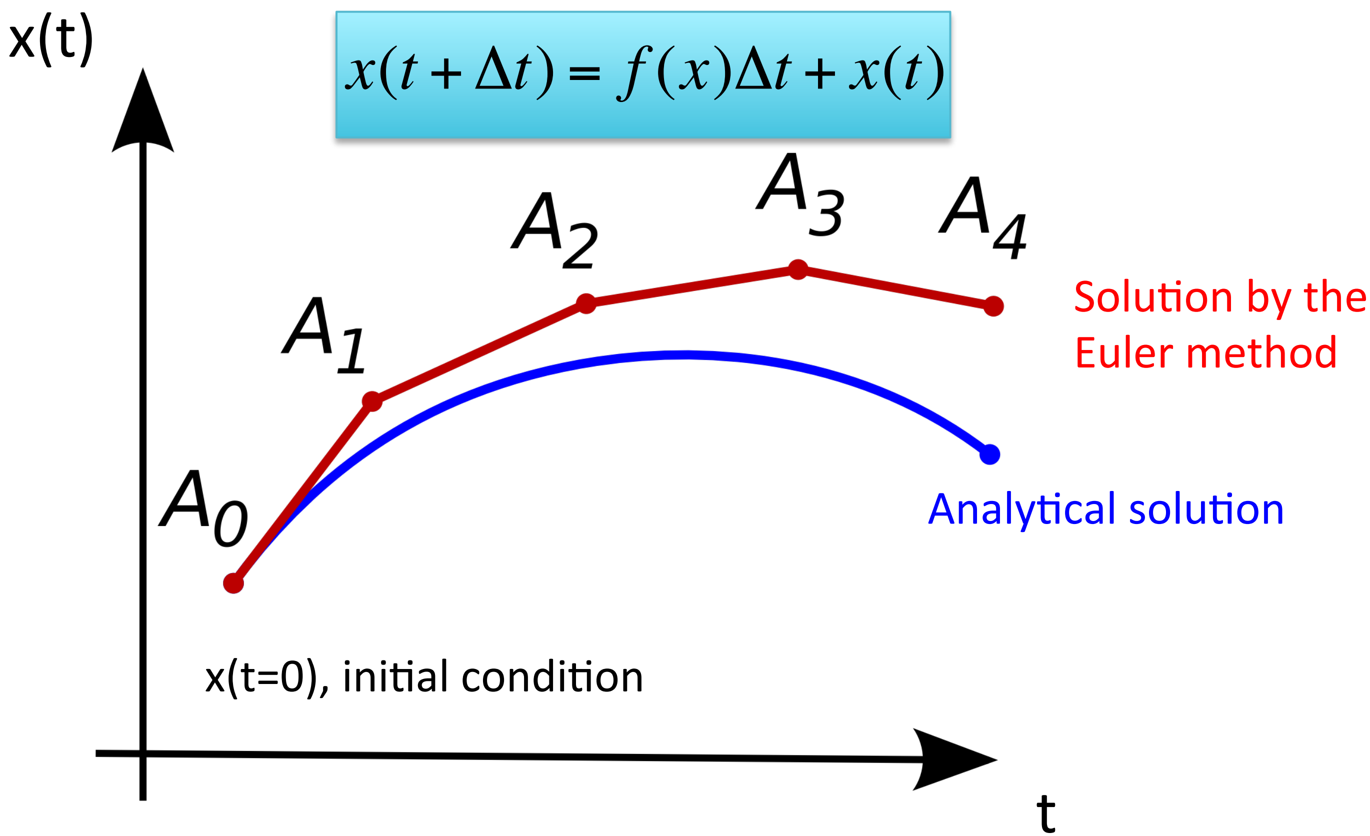
$$x(t+\Delta t) - x(t) = f(x)\Delta t$$

$$\frac{dx}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$x(t+\Delta t) = f(x)\Delta t + x(t)$$

Euler method

Euler method of integration of the differential equation



For details see

http://www.youtube.com/watch?v=RGtCw5E7gBc

Integration with the noise term

$$\frac{dx}{dt} = \sigma\xi(t)$$

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} = \sigma\xi(t)$$

$$x(t+\Delta t) = x(t) + \sigma\xi(t)\cancel{\Delta t}$$

$$x(t+\Delta t) = x(t) + \sigma\xi(t)\sqrt{\Delta t}$$

because discretization adds additional variance
do not forget about multiplication by $\sqrt{\Delta t}$ the noise term

Matlab code for integration of Drift-diffusion model

```
%% Exercise 4-1
m_A = 1;                         % evidence for A
m_B = 0.95;                      % evidence for B
sig = 0.5;                       % noise level

dt = 0.1;                        % time step
T = 200;                         % maximal time
Nt = round(T/dt);                % number of time steps

time=0:dt:T;

x(1) = 0;                        % Initial condition

for k= 2:1:Nt                    % time integration loop

    dxdt = m_A - m_B +sig*randn(1)/sqrt(dt);
    x(k) = x(k-1) + dxdt*dt;

end

plot(time(1:k),x(1:k));

xlabel( 'time t' );
ylabel( 'decision variable x' );
set(gca,'FontSize',20);                     % set the axis with big font
```

$$\frac{dx}{dt} = m_A - m_B + \sigma\eta(t)$$